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By

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NUMERICAL METHODS IN GASDYNAMICS

A. A. Dorodnitsyn

1. Introduction

The advent of high-speed computing machines is of particularly important value in numerical methods in all fields of the exact sciences, among them, gasdynamics, since it allows us to obtain solutions of complete, unsimplified equations with an accuracy that completely satisfies and even exceeds practical requirements. The results have, in many cases, provided much more exact data than a simulated experiment, to say nothing of the fact that the calculation requires much lower expenditures than does an experiment.

The characteristic feature of gasdynamics processes is that, generally speaking, they are accompanied by discontinuities in the functions defining the process (pressure, density, velocity, etc.). Therefore, for the numerical solution of problems in gasdynamics it was necessary to develop mathematical methods which allowed us to obtain discontinuous solutions.

It often happens that a method of solution which is strictly valued for a certain class of functions is actually applicable to a a considerably broader class. Comparatively recently, the hope still existed that the methods of solution of partial differential equations developed for continuous and smooth functions would prove effective for discontinuous solutions or equation coefficients. However, these hopes have not been justified. I shall give one very simple example (suggested by A. A. Samarskiy), which will illustrate how a method which is very good for continuous and smooth functions leads to completely incorrect results when it is applied to discontinuous functions.

Let us consider the simplest one-dimensional equation

$$\frac{d}{dx}k(x)\frac{du}{dx}=0 \tag{1.1}$$

(it may be treated, for example, as a heat-conductivity equation or a one-dimensional discontinuity equation) with boundary conditions

$$u(0)=0, \quad u(1)=1. \tag{1.2}$$

For continuous and smooth $k(x)$ very good numerical solutions are given by the finite-difference scheme

$$k_i\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}+\frac{k_{i+1}-k_{i-1}}{2h}\,\frac{u_{i+1}-u_{i-1}}{2h}=0 \tag{1.3}$$

($\underline{h}$ is the network spacing along $\underline{x}$).

Now let $k(x)$ be a discontinuous function

$$k(x)=\begin{cases}k_1=\text{const} & \text{where} \quad x<\xi,\\ k_2=\text{const} & \text{where} \quad x>\xi,\end{cases} \qquad \frac{k_2}{k_1}=x. \tag{1.4}$$

An accurate solution of (1.1) satisfying the condition of continuity of $u(x)$ (temperature) and of flow $k\,du/dx = 0$ (let us say of heat flow) is easily found

$$(1.5)\qquad \begin{cases} \dfrac{\varkappa x}{1+(\varkappa-1)\xi} & \text{when } x<\xi, \\[2mm] \dfrac{(\varkappa-1)\xi+x}{1+(\varkappa-1)\xi} & \text{when } x>\xi. \end{cases}$$

An exact solution of finite-difference equation (1.4) is also found without difficulty. In fact, let the point of discontinuity ξ lie between x_j and x_{j+1} [if the entire interval (0,1) is broken up into $\underline{n}$ spacings $h = 1/n$, $j/n < \xi < (j + 1)/n$], when $i \leqslant j$

$$u_i = Cx_i$$

(the constant C will be determined later on from the condition $u_n = 1$).

Then, checking the two values u_{j+1} and u_{j+2}, we obtain

$$(1.6)\qquad \begin{cases} u_{j+1} = C\left[x_{j+1} - 2\dfrac{\varkappa-1}{\varkappa+3}h\right], \\[2mm] u_{j+2} = C\left[x_{j+2} - 6\dfrac{(\varkappa-1)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}h\right]. \end{cases}$$

When $i \geqslant j + 2$ Eq. (1.3) again reduces to

$$u_{i+1} - 2u_i + u_{i-1} = 0$$

and when $i \geqslant j + 2$ elementary calculation gives

$$(1.7)\qquad u_i = C\left[\frac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}x_i + \left(1 - \frac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}\right)x_{j+1} + \frac{2(\varkappa-1)(13\varkappa+7)}{(\varkappa+3)(5\varkappa-1)}h\right].$$

Setting $i = n$, and taking into account the fact that $u_n = u(1) = 1$, we obtain for C the expression;

$$C = \frac{1}{\left[\dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)} + \left(1 - \dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}\right)x_{j+1} + \dfrac{2(\varkappa-1)(13\varkappa+7)}{(\varkappa+3)(5\varkappa-1)}h\right]}.$$

By passage to the limit $h \to 0$ ($x_{j+1} \to \xi$) we obtain:

$$C = \frac{1}{\xi + \dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}(1-\xi)}$$

and for u

$$
(1.8)\qquad u=\begin{cases}\dfrac{x}{\xi+\dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}(1-\xi)}, & \text{when } x\leqslant\xi,\\[2ex] \dfrac{\xi+\dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}(x-\xi)}{\xi+\dfrac{(5-\varkappa)(3\varkappa+1)}{(\varkappa+3)(5\varkappa-1)}(1-\xi)}, & \text{when } x\geqslant\xi.\end{cases}
$$

Eqs. (1.5) and (1.8) coincide only when $\varkappa = 1$ (that is, $k_1 = k_2$), thus the solution of finite-difference equation (1.3), even though, generally speaking, it approaches a definite limit (1.8) [only for certain values of ξ and $\varkappa$ can the denominator in Eqs. (1.8) become zero], does not give the solution to the stated problem. As is easily seen there corresponds to the solution of the finite-difference scheme the presence of a heat source at the point $x = \xi$

$$
\left\{\left[k\frac{du}{dx}\right]_{x=\xi+0}-\left[k\frac{du}{dx}\right]_{x=\xi-0}=-\frac{3k_1(\varkappa-1)^2}{(5-\xi)(3\varkappa+1)+8(\varkappa^2-1)\xi}\right\},
$$

i.e., the finite-difference scheme does not ensure the integral condition of conservation of heat flow at points of discontinuity.

It should be noted that the property of finite-difference schemes observed in this example forces us to pay very close attention to their use. In the solution of nonlinear gasdynamics equations, we are seldom successful in rigidly proving that the method converges to the required solution. The convergence is usually verified "empirically," by carrying out a series of calculations with ever decreasing network spacings, and if, beginning with some value of h a further decrease in it does not change the result within the prescribed accuracy of the calculation, we usually accept the convergence "on faith".

The example just presented shows that in the case of discontinuous solutions this criterion of the correctness of the method leads to error.

The source of error is the fact that the finite difference scheme chosen for the solution of the equation does not approximate the integral law of conservation of heat flow in the presence of discontinuities. Thus we must construct approximating operators so that in the limiting case of discontinuity they will accurately represent the integral laws of conservation.

2. Method of Finite Differences

The method of finite difference is the most common for the solution of partial differential equations. First developed for equations of the elliptic type, it was then generalized for hyperbolic and parabolic equations. In recent years this method has found wide application to the solution of nonlinear partial differential equations, as well as in cases when their solutions are discontinuous. I should like to dwell in detail on the latter problem.

Two methods of approaching the solution of this problem are recognized at the present time. The first method dates back to Richtmayer and consists of the introduction of an "artificial viscosity" into the initial equations. Here, therefore, the original problem is not solved, but a modified physical problem, in which the discontinuities are excluded. The discontinuities are replaced by regions of abrupt change in values. The solution of the original problem is obtained as the limit of the solution to the modified problem as the viscosity coefficient tends to zero. The second method, that about which we were speaking in the introduction, consists of the special construction of finite-difference schemes for which the integral conservation laws are satisfied in the limiting case of a discontinuity.

I shall dwell only briefly on the artificial-viscosity method, pointing out those difficulties which are encountered in its practical application.

In the numerical solution of equations by the artififial-viscosity method, it is actually necessary to pass to a limit twice: first for a fixed viscosity coefficient the network spacing tends to zero and then (after obtaining a number of solutions with different viscosity coefficients) the viscosity coefficient tends to zero. Of course, this method of calculation will be extremely cumbersome. In reality the calculation is carried out in such a way that the coefficient of viscosity decreases simultaneously with the network spacing (in practice the viscosity coefficient is chosen so that the shock wave will be "spread" over 5-6 network spacings).

The introduction of viscosity generally lowers the accuracy of the calculation. However, the viscosity is needed only where there are shock waves and is not needed where the solution is smooth. Therefore, the viscosity coefficient will not be taken constant, but as a function of the velocity gradient, in the following form.

If the initial equation was

$$\frac{\partial u}{dt} + \frac{\partial F(u)}{\partial x} = 0 \tag{2.1}$$

[u and F(u) may be considered vectors], the modified equation may be written:

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = \varepsilon \frac{\partial}{\partial x}\left\{\Psi\left(\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial x}\right\}, \tag{2.2}$$

where $\Psi\left(\frac{\partial u}{\partial x}\right)$ is a positive function which increases with an increase in the velocity gradient $\partial u/\partial x$ [we may also set $\Psi(0) = 0$]. Such a device accomplishes a decrease in the influence of viscosity where its introduction is disadvantageous.

The other difficulty is the fact that in replacing Eq. (2.2) by a finite-difference equation, convergence is ensured in the presence of discontinuities and when $\overline{\varepsilon} \to 0$ only when this difference equation has first-order accuracy. Therefore, in order to obtain a sufficiently accurate result, it is necessary to carry out the calculation with a very large number of network nodes. The calculation becomes exceedingly cumbersome even for the best high-speed computing machines.

Of course convergence of difference schemes of a higher order of accuracy is disrupted in the vicinity of discontinuities. In regions of smooth solution, finite-difference schemes of a higher order of accuracy improve the convergence. In order to ensure convergence and, at the same time, not impair the convergnece outside the vicinity of the shock wave, finite-difference equations are used, which may conditionally be called "schemes of intermediate accuracy". Let $\mathcal{L}^{(1)}(u) \doteq 0$ be a finite-difference equation of first-order accuracy, approximating Eq. (2.2), and $\mathcal{L}^{(2)}(u) = 0$. a finite difference equation of second-order accuracy. Any equation of the form

(2.3) $$\alpha \mathcal{L}^{(1)}(u) + (1-\alpha) \mathcal{L}^{(2)}(u) = 0$$

will also approximate Eq. (2.2) and if $\alpha > 0$ and fixed, the expression on the right side of (2.3) is of first-order accuracy. But it is possible to take the quantity α as a variable; to take it as equal to or close to unity where there is "danger" of the formation of shock waves, and, conversely, where the solution is smooth to take it as small or equal to zero. Of course, this process of selecting α must be automated for machine calculation, for which it is necessary to compute the "criterion of wave generation" by the values of u at a number of neighboring points and to assign α as some function of this criterion. This method allows us to increase the accuracy of the calculation. However, it leads to a very appreciable complication of the logical scheme of the calculation, that is, it significantly complicates the programming of problems for high-speed computing machines.

Let us now consider the second method of solving discontinuous problems. The differential equations of gasdynamics are expressions of the laws of the conservation of mass, momentum, and energy. Therefore we may list them in "divergent" form.

In the case of two independent variables (two coordinates or one spacial variable and time), the equations may be written in "divergent" form as follows:

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = F, \tag{2.4}$$

where P, Q, and F are some functions of unknown quantities and, perhaps, independent variables. Right sides which differ from zero may occur in the presence of sources (for example heat liberation due to chemical reactions). Integrating Eq. (2.4) over some domain $\mathcal{D}$ bounded by the contour $\mathcal{L}$, we arrive at the integral relation:

$$\int_{\mathcal{L}} P dy - Q dx = \iint_{\mathcal{D}} F dx dy, \tag{2.5}$$

which will be valid not only for continuous P and Q, but for piecewise-continuous as well. Thus equations in the form of (2.5) will always be effective for gasdynamic processes, while Eqs. (2.4) may lose meaning in the vicinity of discontinuities. Specifically, all the conditions for the discontinuities are obtained from Eq. (2.5).

Therefore, even when constructing numerical methods of solution suitable for application to the case of discontinuities, it is necessary to proceed from equations in the form of (2.5), and not from partial differential equations.

For clarity we shall examine the construction of finite-difference equations for one-dimensional plane motion of a gas. By using the Lagrangian coordinate system in which the variables are the time $\underline{t}$ and the mass of a gas column of unit area and cross section $\underline{x}$ (read from some initial point). In this coordinate system, the equations of the problem are written in the form:

(2.6) $$\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=0, \quad \frac{\partial v}{\partial t}-\frac{\partial u}{\partial x}=0, \quad \frac{\partial}{\partial t}\left(E+\frac{u^2}{2}\right)+\frac{\partial pu}{\partial x}=0$$

(here $\underline{u}$ is the velocity; $\underline{p}$ the pressure; $\underline{v}$ the specific volume; and E the internal energy, for an ideal gas $E=pv/(\varkappa-1)$, , where $\varkappa=C_p/C_v$ is the ratio of the specific heats).

Corresponding to these differential equations are the integral relations:

(2.7) $$\oint_{\mathfrak{L}}(udx-pdt)=0, \quad \oint_{\mathfrak{L}}(vdx+udt)=0, \quad \oint_{\mathfrak{L}}\left[\left(E+\frac{u^2}{2}\right)dx-pudt\right]=0.$$

After taking a network rectangle with vertices

$$(x_m,\ t_m),\ (x_{m+1},\ t_n),\ (x_{m+1},\ t_{n+1}),\ (x_m,\ t_{m+1})$$

over the contour $\mathfrak{L}$, the integral relations (2.7) may be presented in the form:

(2.8) $$\begin{cases} u^n_{m+\frac{1}{2}}h-P^{n+\frac{1}{2}}_{m+1}\tau-u^{n+1}_{m+\frac{1}{2}}h+P^{n+\frac{1}{2}}_{m}\tau=0, \\ v^n_{m+\frac{1}{2}}h+v^{n+\frac{1}{2}}_{m+1}\tau-v^{n+1}_{m+\frac{1}{2}}h-v^{n+\frac{1}{2}}_{m}\tau=0 \\ \left(E+\frac{u^2}{2}\right)^n_{m+\frac{1}{2}}h-P^{n+\frac{1}{2}}_{m+1}v^{n+\frac{1}{2}}_{m+1}\tau-\left(E+\frac{u^2}{2}\right)^{n+1}_{m+\frac{1}{2}}h+P^{n+\frac{1}{2}}_{m}v^{n+\frac{1}{2}}_{m}\tau=0. \end{cases}$$

Here $h=x_{m+1}-x_m,\ \tau=t_{n+1}-t_n$, , the subscript $\underline{m}$ denotes the value of the function when x = x_m, the subscript $\underline{n}$ the value of the function when $t = t_n$, the subscript $m + 1/2$ some mean value in the interval (x_m, x_{m+1}) and $n + 1/2$ some mean value in the interval (t_n, t_{n+1}).

If all quantities were continuous, we could easily relate the mean values of the quantities to the values at the nodes of the network (for example, by taking the half-sum of neighboring nodal values). However, since we wish to construct a method of calculation which is also suitable in the presence of shock waves, we should consider any point in the region a possible point of discontinuity. In the finite-difference interpretation, this means every network node should be considered a location of discontinuity. Thus at the moment of time t_n we consider the point x_m a point of discontinuity to the right of which the values of S are equal to $S^n_{m+1/2}$, and to the left, equal to $S^n_{m-1/2}$. Then waves (shock or rarification) will leave in both directions from thepoint x_m, at that same point x_m values of the quantities will be established corresponding to the integral conditions for the discontinuity $[cu - p] = 0$, $[cv + u] = 0$, $[c(E + u^2/2) - pu] = 0$, where $\underline{c}$ is the velocity of the shock wave. This value will be retained until waves from other network nodes reach this point. But if the time spacing τ is chosen so that this does not occur (this condition also satisfies the convergence condition of the finite-difference method for hyperbolic equations), the quantities P_m, v_m etc. will remain constant throughout the entire interval.

The dependences between P_m, $P_{m+1/2}$, and $P_{m-1/2}$ may be presented (after transforming the conditions for discontinuities in the form

(2.9)
$$\begin{cases} c_{л}^2 = \dfrac{1}{2v_{m-\frac{1}{2}}}\left[(\varkappa+1)P_m + (\varkappa-1)P_{n-\frac{1}{2}}\right], \\ c_n^2 = \dfrac{1}{2v_{m+\frac{1}{2}}}\left[(\varkappa+1)P_m + (\varkappa-1)P_{m+\frac{1}{2}}\right], \\ P_m(c_{л} + c_n) = c_n P_{m-\frac{1}{2}} + c_{л} P_{m+\frac{1}{2}} + c_{л} c_n (u_{m-\frac{1}{2}} - u_{m-\frac{1}{2}}). \end{cases}$$

Here c_l and c_r are the absolute values of the velocities of the left and right shock waves respectively. Eqs. (2.9) are valid if both shock waves $(P_m > P_{m-\frac{1}{2}}, P_m > P_{m\,\frac{1}{2}})$. For rarification waves, Eqs. (2.9) should be replaced by the corresponding conditions for rarification waves. After determining P_m from Eq. (2.9), v_m is found elementarily

$$v_m(c_n + c_l) = c_l u_{m-\frac{1}{2}} + c_n u_{m+\frac{1}{2}} - P_{m+\frac{1}{2}}$$

and Eqs. (2.8) allow us to make the time spacing.

The method set forth above was proposed by S. K. Godunov in 1952 and has been successfully applied to the solution of practical problems.

In constructing a finite-difference approximation for partial differential equations to obtaining discontinuous solutions, each network node must be considered a possible point of discontinuity. The relationship between the quantities for the discontinuity must be taken in accordance with the integral conditions of conservation (i.e. to take these relations as exact, as in the example set forth above, or approximate, but such as to ensure the prescribed accuracy of calculation).

3. Method of Integral Relations

A very effective method for solving problems on high-speed computing machines is the method of approximate reduction of partial differential equations to systems of ordinary differential equations. This method allows us to make use of the highly developed apparatus for the solution of ordinary differential equations, and, in addition, in the presence of unbounded domains it allows us to use other well developed methods for the asymptotic solution of ordinary differential equations.

Difficulties arise in the application of this method (especially with nonlinear equations) when it is necessary to solve a boundary-value problem for the approximate systems of ordinary differential equations. Therefore, this method is effective for boundary-value problems in those cases which allow us to find a sufficiently accurate solution when the order of the approximate system of ordinary equations is low. As experience has shown, a highly satisfactory accuracy is attained when integral methods are used, i.e., integral methods when the integral relations expressing the laws of conservation are set as the basis for construction of an approximate system of ordinary differential equations.

As stated above, the gasdynamics equations may be presented in "divergent" form:

$$\frac{\partial}{\partial x} P_i(x, y, u_1, \ldots, u_n) + \frac{\partial}{\partial y} Q_i(x, y, u_1, \ldots, u_n) = F_i(x, y, u_1, \ldots, u_n) \tag{3.1}$$

$$(i = 1, 2, 3, \ldots, n),$$

where $u_1, \ldots, u_n$ are unknown functions.

Let it be necessary to solve system of equations (3.1) in domain $\mathcal{D}$, which has the shape of a curvilinear rectangle, the boundaries of which, let us say, are the lines

$$x = a, \; x = b, \quad y = 0, \; y = \delta(x) \tag{3.2}$$

(in special cases a may be equal to $-\infty$, $b = +\infty$). Dividing the domain up into N strips of width δ/N and integrating each of the equations of system (3.1) with respect to y across each strip, we obtain the system of integral relations

$$\frac{d}{dx} \int_{y_k}^{y_{k+1}} P_i dy - \frac{\delta'}{N} [(k+1) P_{i,k+1} - k P_{i,k}] + Q_{i,k+1} - Q_{i,k} = \int_{y_k}^{y_{k+1}} F_i dy, \tag{3.3}$$

$$i = 1, 2, \ldots, n; \quad k = 0, 1, \ldots, N-1; \quad y_k = \frac{k}{N} \delta.$$

The upper boundary of the domain $y = \delta(x)$ can be a given function, and it can also be an unknown function, subject to determination. In the first case a total of n boundary conditions should be given at the boundaries $y = 0$ and $y = \delta(x)$, and in the second case, $n + 1$ boundary conditions should be given.

Of course the lower boundary of the domain may also be curvilinear. The generalization of integral relations (3.3) for this case is obvious.

Integral relations (3.3) present themselves physically as the conservation laws (mass, energy, momentum, etc.) set down for the strips.

If we now represent the functions P_1 and F_1 with the aid of some interpolation formulas by their values $P_{1,k}$ and $F_{1,k}$ at the boundaries of the strips, the integrals in the relations (3.3) are approximately represented in the form

$$\int_{y_k}^{y_{k+1}} P_i dy = \delta \sum_{\nu=0}^{N} A_{k,\nu} P_{i,\nu} \tag{3.4}$$

and analogously for the integral of F_1. In Eq. (3.4) the coefficients $A_{k,\nu}$ are definite numbers which depend on the chosen interpolation formula.

The substitution of Eq. (3.4) into integral relations (3.3) leads to a system of nN ordinary differential equations relative to $n(N + 1)$ unknown functions $u_{1,k}$ (or $n(N + 1) + 1$ unknown functions if $\delta(x)$ is also previously unknown). The addition to the system of n (or $n + 1$ when $\delta(x)$ is not given) conditions for the boundaries $y = 0$ and $y = \delta(x)$ closes the system, and thus the problem is reduced to a closed system of ordinary differential equations.

Note that the selection of the system of functions, with which the interpolation formulas for P_1 and F_1 are constructed, has significant importance for the accuracy of the calculation. The "inaptness" of the selection of the system of interpolation functions depends, of course, on how well we represent qualitative behavior of the solution.

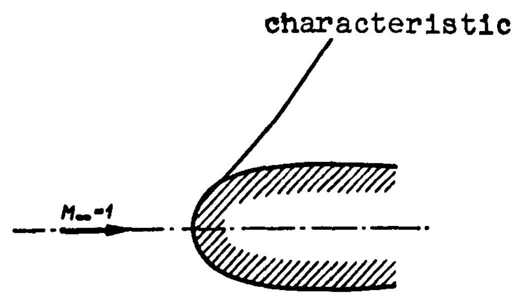

Fig. 1. Flow-past diagram at $M_\infty = 1$.

As is apparent from the above, the method of integral relations is easily applied to the case of unknown boundaries of a domain. This has special significance in gasdynamics problems, where the shape of the shock wave and the boundaries of the domains of influence are not known beforehand.

Let us consider, for example, the problem of flow past a body with a velocity exactly equal to the speed of sound. This problem was solved by P. I. Chushkin. For simplicity, we shall consider a symmetrical body. Here it is necessary to find a solution to the equations of gasdynamics in the domain bounded by the axis of symmetry, the body contour, and the characteristic of the first family tangent to the sound curve at infinity (Fig. 1). The shape of this characteristic is unknown. Corresponding to the line $y = 0$ in the general theory of the method is the axis of symmetry; corresponding to the curve $y = \delta(x)$ is the above-mentioned characteristic. We have two relations for this characteristic: the differential equation of the characteristic itself and the relation between the increments in the dope of the velocity vector and the increments in the velocity itself.

In this problem the number of unknown functions $\underline{n}$ is equal to 2 (let us say the velocity $\underline{v}$ and the slope of the velocity ϑ). For the axis of symmetry we have one condition ($\vartheta = 0$), and 2 conditions for the unknown characteristic, a total therefore of $3 = n + 1$ conditions.

In the problem of the supersonic flow past a blunt body, i.e., with a detached shock wave, the shape of the shock wave is unknown. Corresponding to the shock wave is the boundary $y = \delta(x)$ of the general theory; corresponding to the body contour is the boundary $y = 0$ (Fig. 2). In this problem there are a total of 4 unknown functions (e.g., the two components of the velocity $\underline{u}$ and $\underline{v}$, the density ρ, and the pressure $\bar{p}$). The condition of no flow by the body contour provides one boundary condition, and for the shock wave we have four conditions (each of the quantities $\underline{u}$, $\underline{v}$, ρ, $\underline{p}$ expressed in terms of the slope of the shock wave). Altogether, therefore, we have $5 = n + 1$ conditions, i.e., as many as are necessary for the construction of a closed system of ordinary differential equations, in which the width of the region will enter as one of the unknown functions. The problem of the supersonic flow past bodies with a detached shock wave was solved by O. M. Belotserkovskiy.

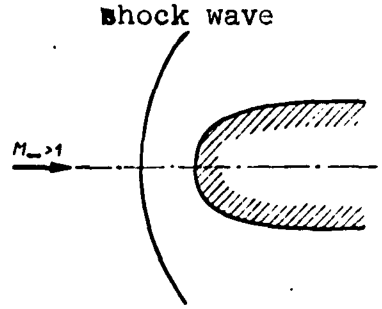

Fig. 2. Flow-past diagram with detached shock wave.

The two problems given here, as far as I know, have not been solved by any other method.

It is interesting to illustrate the actual rapidity of convergence of the method of integral relations using the results of P. I. Chuskin and O. M. Belotserkovskiy. Calculations of the critical Mach number for ellipses and ellipsoids according in the first, second, and third approximations are presented in Fig. 3. The velocity distribution around a circle when $M_\infty = M_{cr}$ in the presence of circulation is given in Fig. 4.

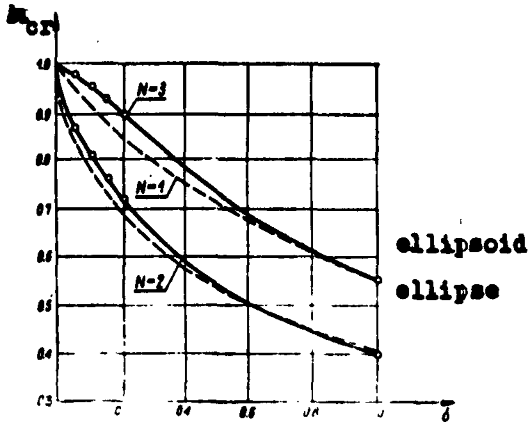

Fig. 3. Critical Mach number distribution for ellipses and ellipsoids.

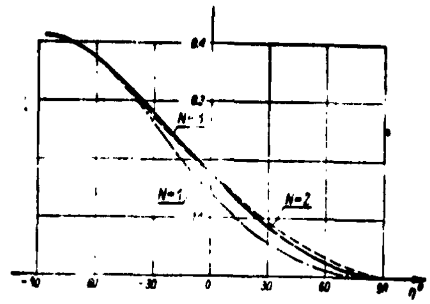

Fig. 4. Velocity distribution around a circle with circulation at M_{cr}.

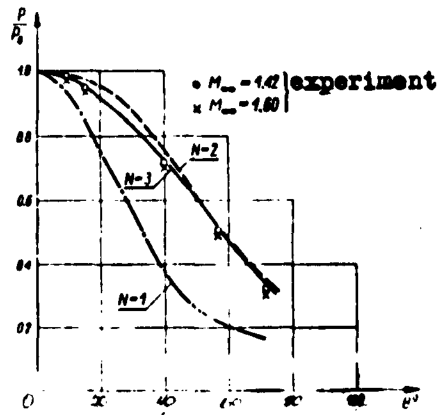

Fig. 5. Pressure distribution around a circle when $M_\infty = 1$.

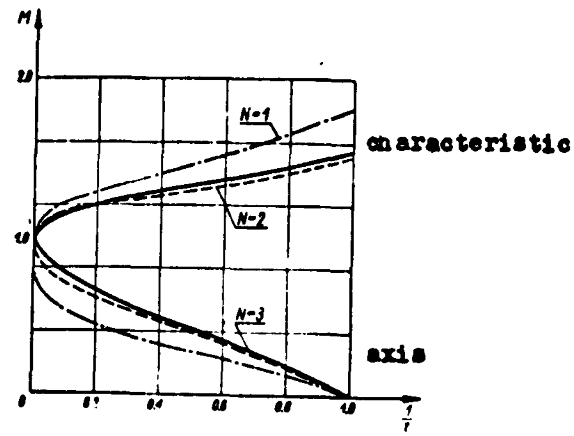

Fig. 6. Distribution of M_∞ along the limiting characteristic and along the axis for a circle when $M_\infty = 1$.

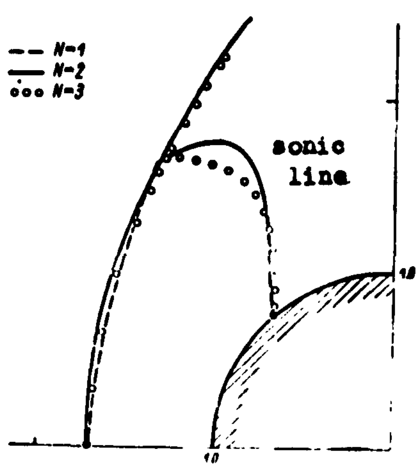

Fig. 7. Shape of the shock wave for a circle when $M_\infty = 3$.

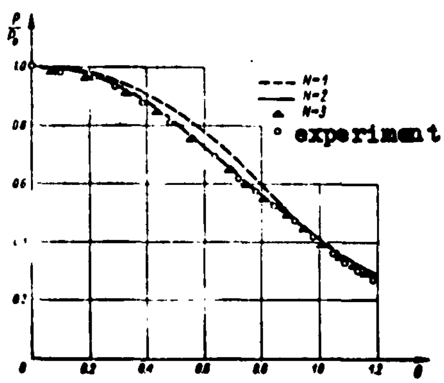

Fig. 8. Pressure distribution around a circle when $M_\infty = 3$.

In Fig. 5 and 6 are listed the results of calculations of the flow past a circle of a gas stream at the speed of sound at infinity.

The shape of the shock wave is shown in Fig. 7, and the pressure distribution around a cylinder with flow past by a supersonic stream is shown in Fig. 8.

As is apparent from the graphs, the calculation is sufficiently accurate in the second approximation, in which the entire domain has been subdivided into only two strips. We can hardly say here that the law of the convergence to zero of the error of the approximation appeared in the second or third approximation. From a practical point of view, the limiting law of convergence to zero of the error of the method is of little interest. It is important for the method that the accuracy needed in practice be attained in low-order approximations. And the examples presented here illustrate the effectiveness of the method of integral relations in this respect.

4. Remark on the Method of Characteristics

The classical problems of the method of characteristics — Cauchy's problem, Goursat's problem, and also the determination of the gas flow along a given characteristic curve of the first family and the contour of the body past which the flow occurs — at the present time (with calculation on high-speed computing machines) can be considered extremely simple problems.

Cases in which characteristic curves of one family intersect, i.e. in which shock waves arise, present the greatest difficulty when the method of characteristics is used.

In steady-state problems of aerodynamics, it is possible in most cases to determine the presence of shock waves and their approximate position beforehand. This is considerably harder to do in nonsteady-state problems. Therefore the method of finite differences is chiefly used in the solution of nonsteady-state problems, though the convergence of the method of characteristics is considerably better than that of the method of finite differences.

Known difficulties are encountered when the method of characteristics is used in the vicinity of the sound curve and when constructing a head-attached shock wave.

In the first case it is most advisable to depart from the sound curve with the aid of series (it is, in practice, sufficient to use one or two terms of the series).

In calculating by the method of characteristics with simultaneous construction of the shock wave, it is difficult to ensure stability of the calculation, especially if the angles of intersection of the characteristic curves of the first family with the shock wave are small. Here it is also more advisable to use series in the construction of the initial element of the shock wave.

5. Conclusion

In conclusion, I wish to note that from my point of view the most important problem at the present time in the field of the numerical methods in gasdynamics is the development of effective methods for the solution of three-dimensional problems (spatial steady-state flows and nonsteady-state two-dimensional ones).

Of course, one can not say that all two-dimensional problems have already been solved and that nothing remains to be done with regard to improving the methods of calculation. However, one can say that with high-speed computing machines almost any two-dimensional problem can be solved, even if in a very cumbersome way. Although only relatively simple three-dimensional problems are being solved, there still remains in this problem many fundamental difficulties, and not only computation difficulties. As regards the practical significance of three-dimensional problems, this is obvious to everyone.

The development of methods for the numerical solution of three-dimensional problems would give scientists and engineers in the field of aerodynamics a powerful tool in their scientific research, as well as in the solution of actual engineering problems.

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